Mutual enhancement of magnetism and Fulde-Ferrell-Larkin-Ovchinnikov superconductivity in $CeCoIn_5$

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Recent experiments on $CeCoIn_5$ suggest an unusual interplay between superconducting and magnetic orders that gives rise to a multicomponent (magnetosuperconducting) phase. We demonstrate that characteristics of $CeCoIn_5$ make this system particularly well suited for the onset of such a phase. Based on general considerations, we show that superconductivity with nonzero Cooper-pair momentum may lead to an enhancement of the spin-spin response function and, simultaneously, incommensurate spin-density wave may enhance the Cooper-pair susceptibility.

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I. INTRODUCTION

It was predicted in the middle of the 1960s that unusual superconducting state with nonvanishing momentum of the Cooper pairs may occur at low temperatures and in strong magnetic fields.[1,2] This state has recently been analyzed in the context of heavy fermion systems,[3–16] organic superconductors,[17,18] ultracold atoms,[19,20] and a dense nuclear matter.[21–23] Despite intensive search, for many years physicists failed to find a direct experimental evidence for the existence of the Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) superconductivity. This lack of success has commonly been attributed to very sever requirements for the formation of the FFLO state. Experimental results reported a few years ago indicated that all the theoretical requirements are satisfied by the heavy fermion superconductor $CeCoIn_5$.[24,25] In particular, it is a clean superconductor[26] with strongly two-dimensional Fermi surface[27] and high critical field that approaches the Pauli limit.[28] Consequently, the specific heat anomaly observed within the superconducting state was recognized as a transition between the Bardeen-Cooper-Schrieffer (BCS) and FFLO phases.[24] Recently reported NMR data indicate the presence of the static spin moments in the high-field regime.[29] It raised the question, whether the specific heat anomaly originates from the onset of magnetic order or from the BCS-FFLO transition.

The high-field neutron diffraction experiments have provided clear evidence for the presence of incommensurate spin-density wave (SDW) but, simultaneously, revealed an unexpected result: namely, at the upper critical field magnetic order vanishes simultaneously with superconductivity.[30] Although a nontrivial interplay between superconductivity and magnetism has been expected in strongly correlated systems, long-range superconducting and magnetic orders are usually recognized as competing phenomena. One of recent examples is $CeRhIn_5$, where antiferromagnetic and superconducting phases coexists.[31] However, in the case of $CeCoIn_5$, the antiferromagnetic order occurs only inside the boundaries of the superconducting phase indicating that instead of competition one observes a mutual stabilization of these phases. Understanding of the underlying mechanism is important for clarifying the nature of the high-field superconducting phase of $CeCoIn_5$.[32–34]

From among various types of unconventional superconductivity, the FFLO phase seems to be particularly well suited for the coexistence with other phases. Vanishing of the superconducting order parameter at certain regions of the real space gives way to other orders. However, such inhomogeneity of the superconducting phase restricts applicability of advanced theoretical approaches developed for a study of strongly correlated systems. As a result, inhomogeneous superconductors are commonly studied by means of the Bogoliubov–de Gennes (BdG) equations. This mean-field approach is suitable also for systems with coexisting phases.[35] As in all self-consistent approaches, one has to assume particular interactions responsible for the occurrence of ordered phases. It is a nontrivial problem for many unconventional superconductors, since the pairing mechanisms are still at debate. The quantitative results obtained from a self-consistent solution of the BdG equations are of physical importance provided the mean-field relation between the pairing interaction and the order parameter holds true. However, the latter assumption may not be satisfied in strongly correlated superconductors.

In the present paper, we want to partially overcome these limitations and make our analysis general, up to some degree independent of the details of the underlying microscopic mechanism. In order to achieve this goal we proceed in two steps. In the first one, we assume a simple mean-field Hamiltonian describing a system with d-wave superconductivity (BCS or FFLO) without any magnetic interaction and study the influence of the onset of superconductivity on the static spin susceptibility. In the second complementary step we assume a symmetric situation: the system is described by a nonsuperconducting Hamiltonian including only magnetic correlations and we study the influence of the onset of the magnetic order on the Cooper-pair susceptibility. Certainly, the effective Hamiltonian of $CeCoIn_5$ should include terms responsible for both magnetism and superconductivity. In the normal state the pair susceptibility of a system with pairing interaction should increase with increasing the susceptibility calculated for the same system but without the pairing interaction. Similar argument holds true also for the spin susceptibilities calculated with and without the exchange interaction. Therefore, our results, though derived for incomplete Hamiltonians, provide qualitative information on the system described by the full effective Hamiltonian. What is more important, *without specifying the mechanisms responsible for the ordered phases* we can determine whether magnetic and superconducting orders always compete or the presence of one of them may enhance a tendency toward formation of the other one.

II. METHOD

In accordance with the above scheme we start with a discussion how the spin susceptibility is modified by the onset of BCS or FFLO types of superconductivity. The Hamiltonian we study for this purpose reads:

$$H_{\mathrm{SC}} = -t\sum_{\langle i,j\rangle,\sigma} c^{\dagger}_{i\sigma}c_{j\sigma} - \sum_{i,\sigma}[s(\sigma)h+\mu]c^{\dagger}_{i\sigma}c_{i\sigma} + \sum_{\langle i,j\rangle}[\Delta(\boldsymbol{R}_i,\boldsymbol{R}_j)c^{\dagger}_{i\uparrow}c^{\dagger}_{j\downarrow} + \mathrm{H.c.}], \tag{1}$$

where $c^{\dagger}_{i\sigma}$ creates an electron with spin σ at site i, μ is the chemical potential, h is the Zeeman term for static magnetic field, and $s(\uparrow)=1$ and $s(\downarrow)=-1$. We introduce the d-wave superconducting order parameter originating from the nearest–neighbor pairing, i.e., $\Delta(\boldsymbol{R}_i,\boldsymbol{R}_j)$ is nonzero only for neighboring sites $\langle i,j\rangle$ with $\Delta(\boldsymbol{R}_i,\boldsymbol{R}_i+\hat{y})=-\Delta(\boldsymbol{R}_i,\boldsymbol{R}_i+\hat{x}) =\Delta_0\cos(\boldsymbol{R}_i\cdot\boldsymbol{Q}_{\mathrm{SC}})$. The wave vector $\boldsymbol{Q}_{\mathrm{SC}}$ corresponds to the total momentum of Cooper pairs and distinguishes between the d-wave BCS superconductivity for $\boldsymbol{Q}_{\mathrm{SC}}=0$ and the d-wave FFLO state for $\boldsymbol{Q}_{\mathrm{SC}}\neq 0$. In order to discuss the interplay between superconductivity and the SDW order, we calculate the static response function

$$\chi^{zz}(\boldsymbol{q}) = \lim_{\omega\to 0}\frac{-1}{N}\sum_{ij}\exp(i\boldsymbol{q}\cdot\boldsymbol{R}_{ij})\langle\langle S^z_i|S^z_j\rangle\rangle_{\omega}, \tag{2}$$

where $\langle\langle S^z_i|S^z_j\rangle\rangle_{\omega}$ is the retarded Green function and $\boldsymbol{R}_{ij}\equiv\boldsymbol{R}_i-\boldsymbol{R}_j$. Divergence of $\chi^{zz}(\boldsymbol{q})$ for some $\boldsymbol{q}\neq 0$ implies the spin density wave.[36,37] Since the Hamiltonian Eq. (1) describes only the superconducting phase, this quantity remains finite in our case. However, inclusion of the exchange interaction may cause a divergent behavior of the response function. In particular, for the paramagnetic state in the absence of external field, the mean-field analysis of the interacting case[37,38] leads to the magnetic susceptibility in the form $\tilde{\chi}(\boldsymbol{q},\omega) =\chi(\boldsymbol{q},\omega)[1-I(\boldsymbol{q})\chi(\boldsymbol{q},\omega)]^{-1}$, where $I(\boldsymbol{q})$ is the interaction strength and $\chi(\boldsymbol{q},\omega)$ is calculated in the absence of the interaction. Going beyond this approximation one may still expect that the larger $\chi^{zz}(\boldsymbol{q})$ in the absence of electronic correlations is, the weaker interaction is necessary for the transition to the SDW state. Therefore, without specifying the magnetic correlations one can estimate the qualitative influence of superconductivity on the tendency toward the formation of the SDW. Namely, we focus on the relative change of the response function Eq. (2) due to the onset of BCS/FFLO superconductivity:

$$\delta\chi^{zz}(\boldsymbol{q}) = \frac{\chi^{zz}(\boldsymbol{q})-\chi^{zz}_0(\boldsymbol{q})}{\chi^{zz}_0(\boldsymbol{q})}, \tag{3}$$

where $\chi^{zz}_0(\boldsymbol{q})$ is calculated for $\Delta_0=0$. For the sake of completeness we calculate also the transverse spin susceptibility

$$\chi^{+-}(\boldsymbol{q}) = \lim_{\omega\to 0}\frac{-1}{N}\sum_{ij}\exp(i\boldsymbol{q}\cdot\boldsymbol{R}_{ij})\langle\langle S^+_i|S^-_j\rangle\rangle_{\omega} \tag{4}$$

and study its relative modification $\delta\chi^{+-}(\boldsymbol{q})$ defined in analogy to Eq. (3). Although both the response functions are equal in the absence of magnetic field (up to a factor of 2), the may differ in the regime where the FFLO phase is expected.

We calculate the response functions in two subsequent steps. First, we apply the transformation

$$c_{i\sigma} = \sum_n [u_{in\sigma}\gamma_{n\sigma} - s(\sigma)v^*_{in\sigma}\gamma^{\dagger}_{n\bar{\sigma}}], \tag{5}$$

where the functions $u_{in\sigma}$ and $v_{in\sigma}$ fulfill the BdG equations

$$\sum_j \begin{pmatrix} H_{ij\sigma} & \Delta^*_{i,j} \\ \Delta_{i,j} & -H^*_{ij\bar{\sigma}} \end{pmatrix}\begin{pmatrix} u_{jn\sigma} \\ v_{jn\bar{\sigma}} \end{pmatrix} = E_{n\sigma}\begin{pmatrix} u_{in\sigma} \\ v_{in\bar{\sigma}} \end{pmatrix}. \tag{6}$$

Here, $\Delta_{ij}=\Delta(\boldsymbol{R}_i,\boldsymbol{R}_j)$, $H_{ij\sigma}=-t\delta_{\langle i,j\rangle}-[s(\sigma)h+\mu]\delta_{ij}$, $\delta_{\langle i,j\rangle}=1$ for neighboring sites i,j and vanishes otherwise. Then, making use of Eq. (5) we express $\langle\langle S^+_i|S^-_j\rangle\rangle_{\omega}$ and $\langle\langle S^z_i|S^z_j\rangle\rangle_{\omega}$ as linear combinations of the retarded Green functions $\langle\langle\gamma^{(\dagger)}_{n\sigma}\gamma^{(\dagger)}_{m\mu}|\gamma^{(\dagger)}_{n'\sigma'}\gamma^{(\dagger)}_{m'\mu'}\rangle\rangle_{\omega}$ with two creation ($\gamma^{\dagger}$) and two annihilation (γ) operators. Since the applied transformation leads to a diagonal single-particle Hamiltonian, the latter Green functions can be calculated straightforwardly. In particular, one obtains

$$\begin{aligned}\langle\langle S^z_i|S^z_j\rangle\rangle_{\omega} &= \frac{1}{4}\sum_{mn}[(u^*_{im\uparrow}u_{in\uparrow}+v^*_{im\downarrow}v_{in\downarrow})(u^*_{jn\uparrow}u_{jm\uparrow}+v_{jm\downarrow}v^*_{jn\downarrow})\Xi^{\uparrow\uparrow}_{mn} \\ &\quad + (u^*_{im\uparrow}v^*_{in\uparrow}-v^*_{im\downarrow}u^*_{in\downarrow})(u_{jm\uparrow}v_{jn\uparrow}-v_{jm\downarrow}u_{jn\downarrow})\Xi^{\uparrow\downarrow}_{mn} \\ &\quad + (v_{im\uparrow}u_{in\uparrow}-u_{im\downarrow}v_{in\downarrow})(v^*_{jm\uparrow}u^*_{jn\uparrow}-u^*_{jm\downarrow}v^*_{jn\downarrow})\Xi^{\downarrow\uparrow}_{mn} \\ &\quad + (u_{im\downarrow}u^*_{in\downarrow}+v_{im\uparrow}v^*_{in\uparrow})(v^*_{jm\uparrow}v_{jn\uparrow}+u^*_{jm\downarrow}u_{jn\downarrow})\Xi^{\downarrow\downarrow}_{mn}],\end{aligned} \tag{7}$$

with

$$\Xi^{\sigma\sigma'}_{mn} \equiv \frac{f[s(\sigma)E_{m\sigma}]-f[s(\sigma')E_{n\sigma'}]}{\omega+s(\sigma)E_{m\sigma}-s(\sigma')E_{n\sigma'}}, \tag{8}$$

where f is the Fermi function and $\sigma,\sigma'=\uparrow,\downarrow$. For the sake of brevity, we do not present explicit formula for $\langle\langle S^+_i|S^-_j\rangle\rangle_{\omega}$.

The possible enhancement of the SDW susceptibility alone does not allow to formulate a statement on a mutual stabilization of SDW and superconducting orders. One needs to show that also the SDW phase enhances or, at least, remains neutral for the superconducting correlations. In order to verify this possibility we follow the discussion in Ref. 39 and use similar method of reasoning to that has been applied for the static spin susceptibility. Namely, we investigate the static pair susceptibility in the presence of SDW for a nonsuperconducting system. In this way, without specifying the pairing interaction we estimate the influence of SDW on the tendency toward formation of the Cooper pairs. The system under consideration is modeled by the Hamiltonian

$$H_{\mathrm{SDW}} = -t\sum_{\langle i,j\rangle,\sigma} c^{\dagger}_{i\sigma}c_{j\sigma} - \sum_{i,\sigma}\{s(\sigma)[h + M(\boldsymbol{R}_i)] + \mu\}c^{\dagger}_{i\sigma}c_{i\sigma}, \tag{9}$$

with a site-dependent magnetization $M(\boldsymbol{R}_i) = M_0\cos(\boldsymbol{R}_i\cdot\boldsymbol{Q}_{\mathrm{SDW}})$. This Hamiltonian can be diagonalized with help of a unitary transformation $c_{i\sigma}=\Sigma_n u_{in\sigma}a_{n\sigma}$, where u diagonalizes the Hermitian matrix $H_{ij\sigma}=-t\delta_{\langle i,j\rangle}-\{s(\sigma)[h+M(\boldsymbol{R}_i)]+\mu\}\delta_{ij}$. i.e.,

$$\sum_{ij} u^{*}_{in\sigma}H_{ij\sigma}u_{jm\sigma} = E_{m\sigma}\delta_{mn}. \tag{10}$$

A diagonal form of the one-particle Hamiltonian allows one to calculate the Cooper-pair susceptibility

$$\chi^{\Delta}(\boldsymbol{q}) = \lim_{\omega\to 0}\frac{-1}{N}\sum_{ij}\exp(i\boldsymbol{q}\cdot\boldsymbol{R}_{ij})\langle\langle\hat{\Delta}_i|\hat{\Delta}^{\dagger}_j\rangle\rangle_{\omega}, \tag{11}$$

where $\hat{\Delta}_i \equiv \Sigma_j \eta(\boldsymbol{R}_{ij})c_{i\uparrow}c_{j\downarrow}$. The d-wave pairing symmetry has been introduced through the factor $\eta(\boldsymbol{R})$ that equals 1 (-1) for $\boldsymbol{R}=\pm\hat{x}(\pm\hat{y})$ and zero otherwise. Straightforward calculations lead to

$$\langle\langle\hat{\Delta}_i|\hat{\Delta}^{\dagger}_j\rangle\rangle_{\omega} = \sum_{mn}\sum_{i'j'}\eta(\boldsymbol{R}_{ii'})\eta(\boldsymbol{R}_{jj'})u_{in\uparrow}u_{i'm\downarrow}u^{*}_{j'm\downarrow}u^{*}_{jn\uparrow} \times \frac{f(-E_{n\uparrow})-f(E_{m\downarrow})}{\omega - E_{m\downarrow} - E_{n\uparrow}}. \tag{12}$$

After obtaining the pair susceptibilities in the presence and in the absence of the SDW [$\chi^{\Delta}(\boldsymbol{q})$ and $\chi^{\Delta}_0(\boldsymbol{q})$, respectively], we discuss its relative modification defined in a similar way as in Eq. (3).

III. RESULTS AND DISCUSSION

In the case where superconductivity with nonzero Cooper-pair momentum as well as incommensurate magnetic order are allowed, there is a vast amount of possible states and it is infeasible to present all numerical results in a completely systematic way. Instead, we focus on results for some particular values of $\boldsymbol{Q}_{\mathrm{SDW}}$ and $\boldsymbol{Q}_{\mathrm{SC}}$, for which the effect discussed before does occur. Such an approach is justified in the sense that we show that the described mutual enhancement is possible, but we do not claim that this is a common phenomenon. We restrict our study to a two-dimensional square lattice. Numerical calculations were carried out for a 32×32 lattice with periodic boundary conditions.

First, we demonstrate that the FFLO superconductivity can enhance the tendency toward formation of the SDW order. Panels in Fig. 1 show the relative changes of the spin susceptibility due to the presence of different kinds of superconductivity in the absence of external magnetic field. One

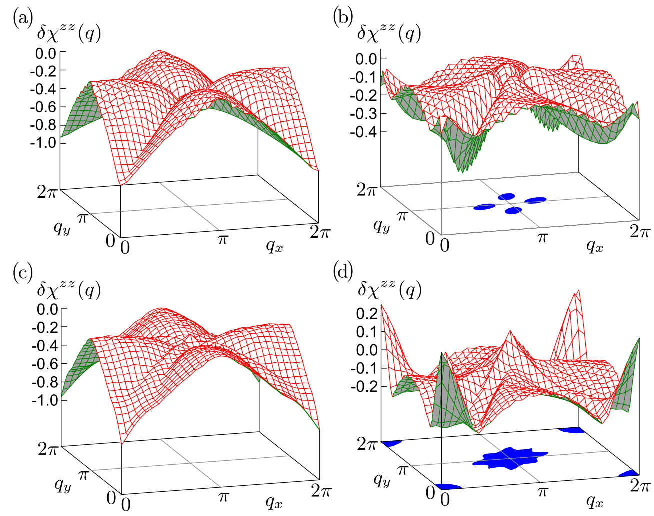

FIG. 1. (Color online) Superconductivity-induced modification of $\chi^{zz}(\boldsymbol{q})$ calculated for $\Delta_0=0.2t$ and $h=0$. The temperature $kT=0.05t$ has been assumed. Panels (a) $(\mu=0)$ and (c) $(\mu=-0.25t)$ show results for d-wave BCS superconductivity. Panels (b) $(\mu=0)$ and (d) $(\mu=-0.25t)$ show results for d-wave FFLO with $\boldsymbol{Q}_{\mathrm{SC}}=(\pi/8,\pi/8)$. Shaded area marks momenta for which $\delta\chi^{zz}(\boldsymbol{q})>0$.

can see that BCS order reduces the susceptibility, whereas in the presence of pairing with a nonzero Cooper-pair momentum the spin susceptibility can be enhanced, at least for some values of $\boldsymbol{q}$. The absence of any enhancement in the BCS phase could explain why this effect, apart from very recent results for $\mathrm{CeCoIn_5}$,[30] has not been observed. As the FFLO phase is stable only close to the upper critical field, we carried out similar calculations for a system in a magnetic field.

The details of $\delta\chi^{zz}(\boldsymbol{q})$ depend on the total momentum of Cooper pairs what can be inferred from a comparison of Figs. 1(d) and 2. However, independently of Q_{SC} superconductivity causes a significant enhancement of the spin susceptibility for $\boldsymbol{q}$ equal or close to (π,π), provided superconductivity is of the FFLO type, i.e., $Q_{\mathrm{SC}}\neq 0$. For some particular values of Q_{SC} there may also be a maximum for $\boldsymbol{q}=\boldsymbol{0}$ [see Fig. 1(d)]. This might suggest that the FFLO phase could enhance ferromagnetism as well. However, as we stated in the Introduction, our approach requires *mutual* enhancement of both the magnetic and superconducting orders. Since, in accordance with the experimental data for $\mathrm{CeCoIn_5}$, we take into account singlet pairing, ferromagnetism would

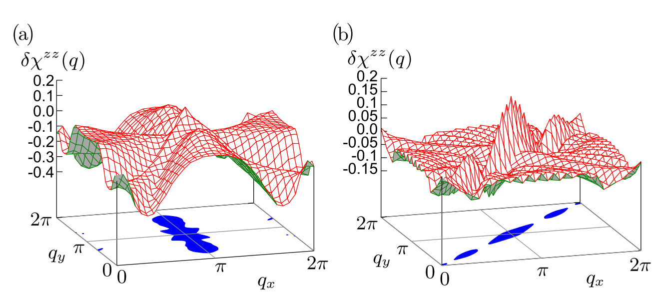

FIG. 2. (Color online) Superconductivity-induced modification of $\chi^{zz}(\boldsymbol{q})$ calculated for the same parameters as in Fig. 1(d). but for $\boldsymbol{Q}_{\mathrm{SC}}=(\pi/8,0)$ (panel a) and $\boldsymbol{Q}_{\mathrm{SC}}=(\pi/2,\pi/2)$ (panel b).

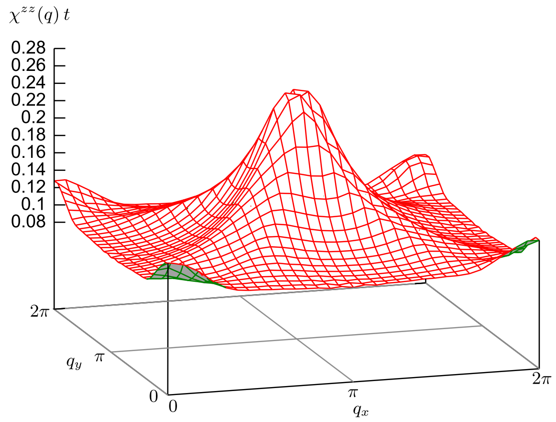

FIG. 3. (Color online) Spin susceptibility $\chi^{zz}(\boldsymbol{q})$ calculated for the same parameters as in Fig. 1(d).

destroy this kind of superconductivity. Moreover, in Figs. 1 and 2 we present only the superconductivity-induced relative change of the spin susceptibility, whereas the susceptibility itself has a maximum for $\boldsymbol{q}$ equal or close to (π,π) (see Fig. 3). Therefore, we focus on the spin susceptibility for the momentum $\boldsymbol{q}=(\pi,\pi)$ and calculate $\delta\chi^{zz}(\pi,\pi)$ and $\delta\chi^{+-}(\pi,\pi)$. These quantities are shown in Figs. 4 and 5, respectively. One can see that superconductivity may significantly enhance the SDW response functions in two regimes: (i) for a low but finite doping, weak magnetic field $h<0.1t$ and nonzero momentum of Cooper pairs [Figs. 4(d) and 5(d)]; (ii) for arbitrary momentum of Cooper pairs but in strong magnetic field $h\simeq t$ and $h>\Delta_0$. In Fig. 6 we show modification of the Cooper pair susceptibility that originates from the onset of incommensurate SDW order for both weak and strong magnetic field. In regime (*i*) the antiferromagnetic order with $\boldsymbol{Q}_{\mathrm{SDW}}=(\pi,\pi)$ strongly reduces the pair susceptibility [see Fig. 7(c)], whereas its reduction due to incommensurate SDW is only of the order of 3% [see Fig. 6(c)]. Therefore, one obtains significant enhancement of the spin-spin correlation function accompanied by only a weak reduction of the pair susceptibility, provided superconductivity is of the FFLO type and the magnetic order is incommensurate SDW. In the regime (ii), the enhancement of response functions is independent of the total momentum of Cooper pairs. However, for $h\simeq t$ the maximum of the Cooper-pair susceptibility occurs for a finite momentum [see Fig. 6(b)] suggesting the FFLO rather that the BCS state. For such a strong magnetic field the maximum of $\chi^{zz}(\boldsymbol{q})$ occurs for $\boldsymbol{q}\neq(\pi,\pi)$, what indicates the tendency toward formation of the incommensu-

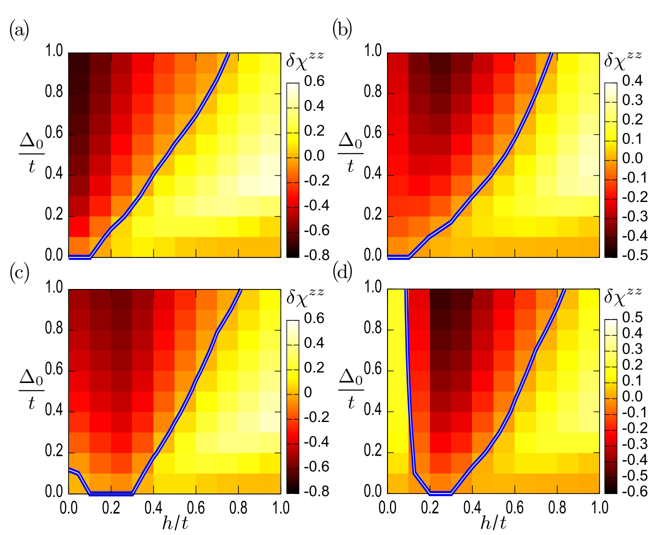

FIG. 4. (Color online) $\delta\chi^{zz}(\pi,\pi)$ calculated for $kT=0.05t$ as a function of h and Δ_0. Panels (a) ($\mu=0$) and (c) ($\mu=-0.25t$) show results for d-wave BCS superconductivity. Panels (b) ($\mu=0$) and (d) ($\mu=-0.25t$) show results for d-wave FFLO with $\boldsymbol{Q}_{\mathrm{SC}}=(\pi/8,\pi/8)$. Superconductivity causes enhancement of the spin susceptibility in the regime that is below the continuous line.

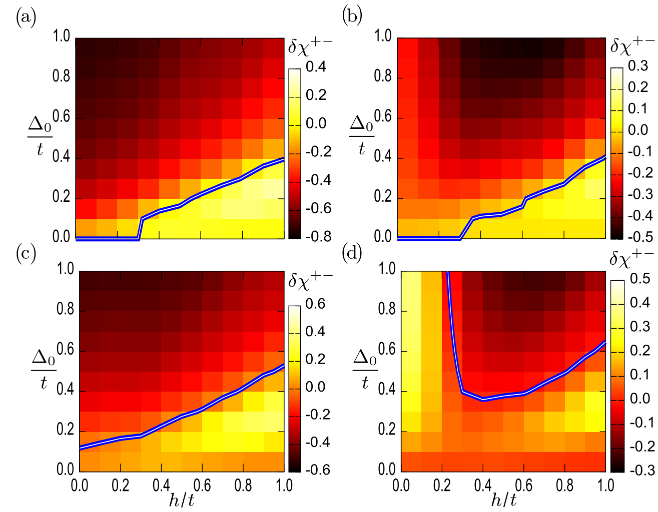

FIG. 5. (Color online) The same as in Fig. 2 but for $\chi^{+-}(\pi,\pi)$ instead of $\chi^{zz}(\pi,\pi)$.

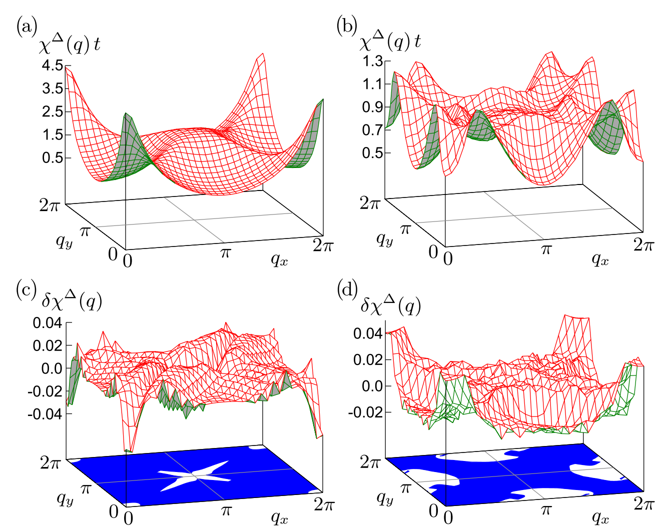

FIG. 6. (Color online) Panels (a) and (b) show the Cooper pair susceptibility $\chi^{\Delta}(\boldsymbol{q})$ in the presence of SDW for $\mu=-0.25t$, $M_0=0.2t$, $kT=0.05t$, and $\boldsymbol{Q}_{\mathrm{SDW}}=(15\pi/16,15\pi/16)$. The magnitudes of magnetic field $h=0.1t$ (a) and $h=0.7t$ (b) have been assumed. Panels (c) and (d) show $\delta\chi^{\Delta}(\boldsymbol{q})$ for the same parameters as panels (a) and (b), respectively. Shaded area in panels (c) and (d) mark momenta for which $\delta\chi^{\Delta}(\boldsymbol{q})>0$.

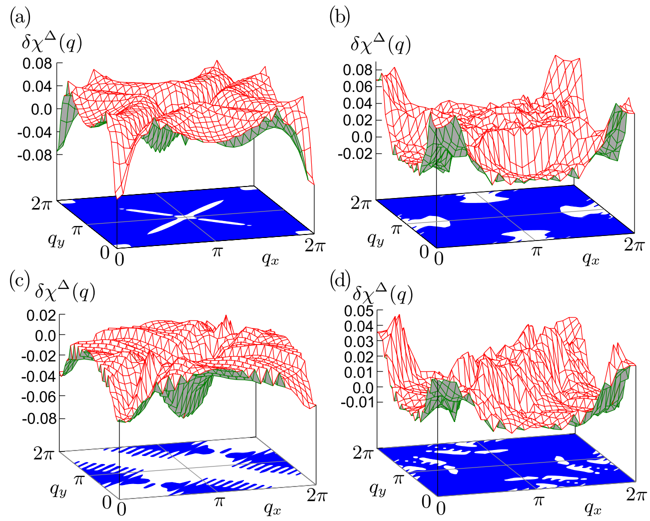

FIG. 7. (Color online) $\delta\chi^{\Delta}(\boldsymbol{q})$ for $h=0.1t$ (left panels) and $h=0.7t$ (right panels). Results in panels (a) and (b) are for $\boldsymbol{Q}_{\rm SDW}=(\pi,\pi)$, whereas in panels (c) and (d) for $\boldsymbol{Q}_{\rm SDW}=(\pi/2,\pi/2)$. The remaining parameters are the same as in Fig. 6.

rate SDW. Therefore, in both the regimes the mutual enhancement concerns the FFLO state and the incommensurate SDW. It does not contradict the well known competition between the BCS superconductivity and the long-range antiferromagnetic (AF) order. As the microscopic origins of superconductivity and magnetism in $CeCoIn_5$ are still under debate, we cannot determine the robustness of FFLO and SDW orders against the magnetic field. Moreover, one may expect that the BCS-like description breaks down in strongly correlated heavy-fermion systems. As a consequence we cannot argue, whether/which regime (*i* or *ii*) may be relevant to the situation observed in $CeCoIn_5$.

The results presented in Figs. 6 and 7 demonstrate how $\delta\chi^{\Delta}(\boldsymbol{q})$ depends on $\boldsymbol{Q}_{\rm SDW}$. Although, the value of $\boldsymbol{Q}_{\rm SDW}$ affects $\delta\chi^{\Delta}(\boldsymbol{q})$ the difference between the results for $\boldsymbol{Q}_{\rm SDW}=(\pi,\pi)$ and $\boldsymbol{Q}_{\rm SDW}=(15\pi/16,15\pi/16)$ is only quantitative. For $h\leq 0.1t$ and $\boldsymbol{q}\ll 1$ the reduction of $\chi^{\Delta}(\boldsymbol{q})$ for $\boldsymbol{Q}_{\rm SDW}=(15\pi/16,15\pi/16)$ is less pronounced than for $\boldsymbol{Q}_{\rm SDW}=(\pi,\pi)$. This results along with the results for the spin susceptibility indicate that the discussed mechanism favors incommensurate SDW with $\boldsymbol{Q}_{\rm SDW}$ close to (but not exactly equal) (π,π).

Finally, we briefly discuss a possible modification of the FFLO, that originates from the onset of SDW. As the latter order exists only within the boundaries of the superconducting phase one might expect that spatial modulation of magnetization is adjusted to that of the superconducting order. However, $\chi^{zz}(\boldsymbol{q})$ is enhanced only for some particular values of $\boldsymbol{q}$ and, consequently, the wave vector of SDW cannot freely adjust to the momentum of Cooper pairs. It has previously been shown that impurities modify the spatial profile of the FFLO superconducting order in such a way that it vanishes (or remains small) in the vicinity of impurities.[40] One may expect that incommensurate SDW should influence the FFLO phase in a qualitatively similar way. It is also worth to recall some results on the vortex structure studied within the hidden-order scenario of high-temperature superconductors.[35] Despite the hidden order occurs only inside the vortex cores, it significantly modifies the vortex profiles. It means that this modification is significant also in the regime where superconductivity dominates. Therefore, we expect that the linear relation between the magnetic field and the momentum of Cooper pairs breaks down in the presence of SDW,[30] however, we cannot determine such a relations within our approach.

Summarizing, we demonstrated that under particular conditions magnetic and superconducting orders can cooperate in the sense that the presence of superconductivity can enhance the tendency toward the formation of magnetic order and the presence of magnetic order can enhance the tendency toward the Cooper pair formation. This mutual enhancement takes place for an incommensurate SDW and FFLO superconductivity, what can be understand as a reflection of the fact that in the presence of an incommensurate SDW there is no symmetry between points $\boldsymbol{k}$ and $-\boldsymbol{k}$, what can favor pairing with a nonzero Cooper pair momentum. The particular conditions may be fulfilled in the FFLO phase of $CeCoIn_5$, where a coupling of these two orders seems to be confirmed.[30] The reported results are, to some extend, independent of the interactions which are responsible for the magnetic and superconducting orders. Although we consider the Bogoliubov quasiparticles, we do not assume the mean-field relations between the pairing interaction, temperature, magnetic field and the order parameters. It is achieved at the expense of a lack of quantitative results. On the other hand, the situation when superconductivity (at least singlet superconductivity) stabilizes the magnetic order is very unusual from both experimental and theoretical point of view. In particular, most of the theoretical analysis based on the Ginsburg-Landau functional lead to a competition rather than cooperation of different orders. However, this conclusion may not hold for systems where SDW coexists with two different superconducting-order parameters.[41] Recently, within the framework of the BdG equations it has been predicted that AF order can coexist with FFLO superconductivity.[42,43] However, we would like to emphasize the difference between our results and those based on the BdG equations. The solutions of the BdG equations show that the AF order occurs in regions, where the superconducting order parameter vanishes. However, there is no macroscopic phase separation and the vanishing of the superconducting order parameter results either from the nature of the FFLO state for a homogeneous case[42] or from inhomogeneities present in the system.[43] These results suggest competition of FFLO and SDW. As opposed to those results, we demonstrate that under tailored conditions these two orders do not compete but mutually enhance each other.

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